Functional thinking in a Year 1 classroom

Elizabeth Warren, Samantha Benson and Sandra Green outline a series of activities that will assist young children understand the key ideas involved in the development of functional thinking.

The concept of a function is fundamental to virtually every aspect of mathematics and every branch of quantitative science. Presently this type of thinking is carolled at the secondary level, and yet it has many benefits for deepening our understanding of early arithmetic. This is particularly so in the way that operations can be considered as 'changing' and how it explicitly illustrates the way in which addition and subtraction are inverse operations, with each 'undoing' the other. With the move to introduce algebraic thinking into the elementary classrooms (e.g., Warren & Cooper, 2005) this paper explores activities that exemplify this thinking with 6 year–old children. The three authors collaboratively planned and implemented a series of hands on activities over an eight lesson program. 'Early algebra' is not the same as 'algebra early'. It is a refocusing of mathematical thinking away from products and towards generalisations of the big ideas in arithmetic within a climate of inquiry and justification (e.g., Brown, 2002). The aim of these learning activities was to assist young children understand the key ideas in this area of mathematics. The activities not only encouraged active learning (Crawford & Witte, 1999) but also reflected the principles of socio-constructivist learning (Vygostky, 1962).

In this context, functions were seen to represent a consistent change between two sets of data. In mathematics, a function is a relation such that each element of a set is associated with a unique element of another set, thus the function rule describes how one value consistently changes into another.

In the early years there are three ideas that children should grasp, namely:

1. Following a function rule to consistently change one set of 'values' into another set of 'values', that is, finding output values when given input values by consistently applying the function rule.
2. Reversing the change process by reversing the rule, that is finding the input values when given the output values by consistently applying the inverse (or reverse) of the function rule; and
3. Identifying the function rule when given a set of input values and a set of output values

Initially these ideas were explored in a 'numberless' world. This reflects a theoretical stance that the structure of mathematics is best explored in a world that does not involve

number as number tends to suggest computation in the minds of many young children (Davydov, 1975) and that changes the focus from how mathematics works to finding answers. For example, the function rule may be as simple as 'adds ***at*** to a letter', 'add ***e*** to the end of a word' or 'cooks the food'. Thus we can even utilise functional relationships in literacy contexts, for example, how do words change if we add an ***'e'*** to the end of them. Which ones make new words? (See Figure 1).

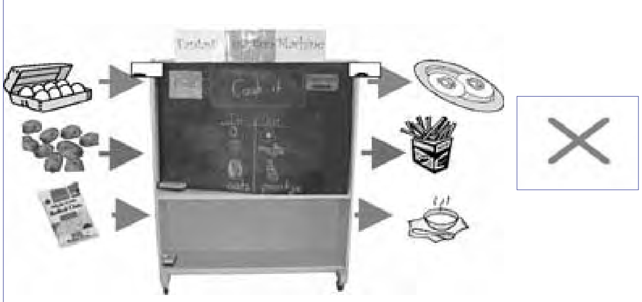


Figure 1:
Developing functional ideas without using numbers

Following the rule:

Input	Change rule	Output
at	Add an e to the end of the word	ate
sit		site
Man		Mane
Hop		
Spit		

Add an ***e*** to the end of the word

Young children enjoy exploring these ideas in play situations where they are actively involved in the process of change. The rule can be attached to the outside of a large box dressed as a function machine that has an input slot and an output slot. As children place words or values in the input slot another child applies the rule and 'posts' the output through the output slot. This type of activity is simply about following the rule. Children are given or create their own input cards. The child in the box changes the card according to the rule and then posts the new card through the output slot. Our research has indicated that such kinesthetic movement is an important dimension of understanding function (Warren & Cooper, 2005).

Reversing the rule

This is an important aspect of functional thinking as it allows us to explicitly explore which operations or processes are related to each other. It also allows us to find unknowns by 'backtracking'. For example, the reverse of 'adding **at**' to a letter is to 'take **at** away' from the end of the word. Thus if we know that the output is hat, what letter did we put into the machine? It is important to show the children that they are 'working backwards' or 'backtracking' by changing the direction of the arrow and reversing the rule. Thus if the rule was 'cook the food' then the reverse would be 'find the raw ingredient' or if the rule was 'flip the shape' the reverse would be 'flip it back'.

Identifying the rule

When given the input set and output set, children were asked to identify the rule that changes one set into the other in 'guess my rule' games. For example, in a numberless world the rule could be 'flip the letters'. In this case the children were given two sets of cards and asked to identify the rule. Children could be given a mirror to help them identify the rule (see Figure 2).

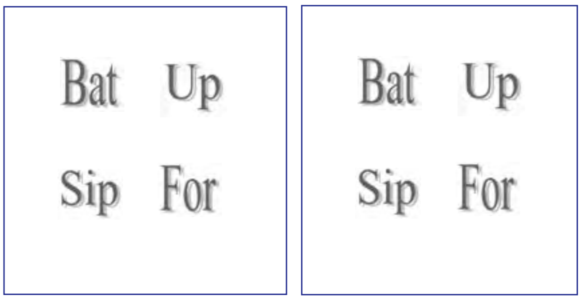


Figure 2:
A mirror is helpful to identify the rule 'flip the letter'

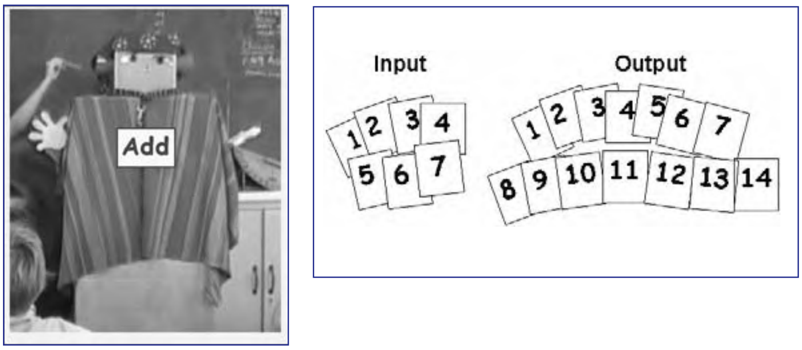


Figure 3:
Mapping these ideas onto number contexts

Mapping these ideas onto number contexts

The main difference between functional thinking and the traditional approach to arithmetic is that we consider 'adding 3' as changing the original number. For example, if I had $4 in a piggy bank and I added another $3 the amount changes to $7. Traditionally we have tended to focus on the operation that 4 and 3 gives 7, a joining notion, rather than 4 changes to 7 by the addition of 3.

Input	Change rule	Output
4		7
2		
5	Add 3 (Count on 3)	
10		
8		

The following representation assists us to emphasise that functions are about changing number by applying a change rule.

+3

Input → Output

Reversing the rule

In this instance functional thinking allows us to explicitly relate the two operations of addition and subtraction, that is, addition is the inverse of subtraction and subtraction is the inverse of addition. Too often at the end of their primary experience many children do not have this understanding and yet it is imperative for solving problems with unknowns. For example, when given a set of output numbers and the change rule of 'add 3' how do you find the numbers we put into the function machine (the input numbers)? ('subtract 3')

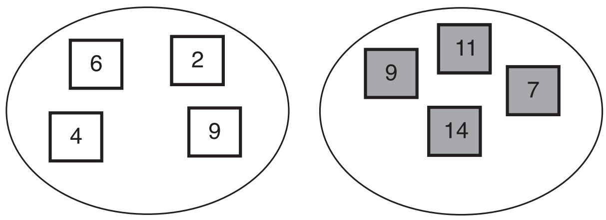
Input	Change rule	Output
		7
		5
	Add 3 (Count on 3)	8

-3

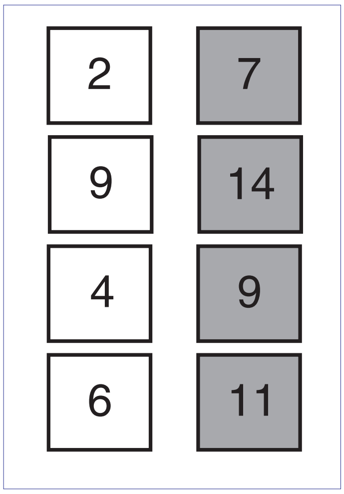
Input ← Output

Identifying the rule

The children were given two sets of cards–one an input set and the other an output set and asked to guess the rule. For example:



An easier task is to give the students the cards with the two sets already aligned.



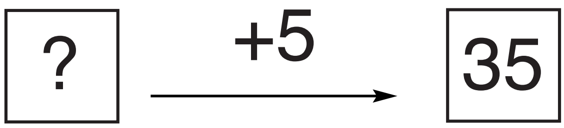
A more challenging task is to provide the students with two sets of cards, ask them to align the cards and find the rule. Remind them that the rule must be the same for each pair.

These activities were not only enjoyed by these young children but also resulted in their looking for functions in other aspects in the world around them. As Samantha and Sandra commented about their students' learning: "They loved the function machine and it made them think outside the square".
"Their whole mental computation and things like that they can visualise a lot more because they visualise it going through the machine and they see it change. Yes they have gained a lot from it".

Where to from here? Using functional thinking to solve for unknowns

Functional thinking is a powerful tool for helping us to solve problems with unknowns. For example, I have some money in my piggy bank. My mother gives me $5 more. I now have $35. How much did I have to start with?

Representing this using arrows



Backtracking

Thus the unknown is $30

In the later years we use this thinking to solve more complex problems and to begin to represent functions in graphical form to help ascertain trends. It also provides answers to questions about various functional situations.

References

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